Total No. of pages: 2 Roll No.: [redacted]

MID-SEMESTER EXAMINATION, September 2025

Course Code: MEMTC301/CEMTC301/CGMTC301/MVMTC301 Max Marks: 20
Course Title: Numerical Methods & Computation Time: 1.5 hours

Note: Attempt all questions in the given order only. Missing data/information (if any) may be suitably assumed & mentioned in the answer.

Q. No.	Question	Marks	CO
1(a)	The system of equations $\begin{cases}\log(x^2+y)-1+y=0,\\ \sqrt{x}+xy=0\end{cases}$ has one approximate solution $(x_0,y_0)=(2.4,-0.6)$. Perform two iterations of the Newton–Raphson method to obtain the root correct to three decimal places.	2	CO1
1(b)	Show that the Gauss–Seidel method for solving the system of equations $\begin{bmatrix}1&1&-1\\2&3&5\\3&2&-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\-6\\4\end{bmatrix}$ diverges.	2	CO1
2(a)	For the system $\begin{bmatrix}3&2&0\\2&3&-1\\0&-1&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5\\4\\1\end{bmatrix}$, find the optimal relaxation parameter ω_{opt} for the SOR iteration scheme. Determine the rate of convergence of this scheme.	2	CO1
2(b)	Using the Jacobi method for symmetric matrix, find all the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix}1&\sqrt{2}&2\\\sqrt{2}&3&\sqrt{2}\\2&\sqrt{2}&1\end{bmatrix}$.	2	CO1

<table>
<tr><td>3(a)</td><td>Find the necessary and sufficient conditions on k so that the Jacobi method converges for solving $Ax = b$.
$$\begin{bmatrix} 1 & 0 & k \\ 1 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}.$$
and b is arbitrary</td><td>2</td><td>CO1</td></tr>
<tr><td>3(b)</td><td>The function $y = \sin x$ is tabulated below:
<table><tr><th>x</th><th>$\sin x$</th></tr><tr><td>0</td><td>0</td></tr><tr><td>$\pi/4$</td><td>0.7071</td></tr><tr><td>$\pi/2$</td><td>1.0</td></tr></table>
Using Lagrange's interpolation formula, find $\sin(\pi/6)$.</td><td>2</td><td>CO2</td></tr>
<tr><td>4(a)</td><td>Using the Gauss forward formula, find $f(32)$ given that
$f(25) = 0.2707,\ f(30) = 0.3027,\ f(35) = 0.3386,\ f(40) = 0.3794.$</td><td>2</td><td>CO2</td></tr>
<tr><td>4(b)</td><td>Obtain the Cubic Spline approximations for the following data
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x)$</td><td>-1</td><td>3</td><td>29</td></tr></table>
with $M_0 = M_2 = 0$ natural spline condition. Hence interpolate at $x = 0.5$</td><td>2</td><td>CO2</td></tr>
<tr><td>5(a)</td><td>Using Newton's backward difference interpolation, interpolate at $x = 1.0$ from the following data:
<table><tr><td>x</td><td>0.1</td><td>0.3</td><td>0.5</td><td>0.7</td><td>0.9</td><td>1.0</td></tr><tr><td>$f(x)$</td><td>-1.699</td><td>-1.073</td><td>-0.375</td><td>0.443</td><td>1.429</td><td>2.631</td></tr></table></td><td>2</td><td>CO2</td></tr>
<tr><td>5(b)</td><td>Show that (i) $\delta = \nabla(1-\nabla)^{1/2}$ and (ii) $\mu = \left(1 + \frac{\delta^2}{4}\right)^{1/2}$.</td><td>2</td><td>CO2</td></tr>
</table>